HEDGING SHORT-TERM INTEREST RISK UNDER TIME-VARYING DISTRIBUTIONS

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INTRODUCTION

The dollar-equivalency approach and the minimum-variance approach are two commonly advocated strategies for hedging the interest rate exposure of debt portfolios. Both strategies have drawbacks which can result in suboptimal hedges. This study investigates the performance of a dynamic hedging strategy employing the GARCH methodology which does not suffer from the limitations of the dollar-equivalency or minimum-variance strategies, and shows that improvements in hedging performance are attainable.

A dollar-equivalency hedge ratio is the ratio of the duration of cash to the duration of futures dollar movements. For zero-coupon, discount instruments this hedge ratio is simply the ratio of the times remaining to maturity for the cash and futures positions which are equal for

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positions in bankers' acceptances and result in an optimal hedge ratio of one. This is referred to herein as the *naive* model. Implicit in the dollar-equivalency approach's grounding in the duration concept is the admission of only small, parallel shifts in the yield curve which implies, strictly speaking, that the prices of bonds and their futures are perfectly correlated. In practice, the prices of bonds and their futures are not perfectly correlated, and so the naive hedge is unlikely to be optimal. This is especially true when the maturity of the hedging instrument does not coincide with the hedging period and when the cash position and the futures contract do not expire at the same time.[1]

The minimum-variance hedging strategy, as its name implies, minimizes the unconditional variance of a hedged portfolio by using a hedge ratio equal to the covariance of cash and futures price changes divided by the variance of futures price changes and is typically estimated by the slope of an ordinary least squares regression of cash on futures price changes [Johnson (1960), Stein (1961), and Ederington (1979)]. This hedging strategy is referred to herein as the OLS model. The main caveat of the OLS model is the assumption that the joint distribution of cash and futures prices is constant and, by implication, that the optimal hedge ratio is time invariant. Evidence exists, however, that cash–futures joint distributions are not always constant. Instability of hedge ratios is reported by Grammatikos and Saunders (1983) for foreign currencies, Hill, Liro, and Schneeweis (1983) for GNMAs, and by Cecchetti, Cumby, and Figlewski (1988) for long-term debt.

There is support for hedging models which admit time variation in the joint distribution of cash and futures prices. Autoregressive conditional heteroscedasticity (ARCH) models introduced by Engle (1982) and generalized by Bollerslev (1986) are proving to be particularly useful in the modeling of time variability of hedge ratios.[2] Park and Bera (1987) find that accounting for ARCH effects when hedging GNMAs with GNMA and T-bill futures results in more efficient hedge ratio estimates, and that hedging efficiency is improved when T-bill futures are used as the hedging instrument. Baillie and Myers (1991) and Myers (1991) report substantial time variation in hedge ratios for various agricultural

[1]The dollar equivalency approach is more common in the professional literature while the duration-based hedging technique is the standard technique presented in academic texts such as Hull (1993) and Stoll and Whaley (1993). The closed-form expressions defining the optimal hedge ratios under each of these two approaches may appear to be different at first blush but both techniques yield exactly the same hedge ratios when applied correctly. See Powers and Castelino (1991) for a description of the dollar equivalency approach.

[2]See Bollerslev, Chou, and Kroner (1992) for a survey of theoretical and empirical developments in the ARCH literature.

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commodities, and Kroner and Sultan (1993) report that hedge ratio estimates accounting for GARCH effects lead to greater risk reduction than conventional or static models.

This study employs GARCH estimation of hedge ratios in order to address the limitations of the naive and OLS hedging models. The use of the GARCH model extends previous research in two respects. The model permits time-variation in the joint distribution of cash and futures prices, and, in particular, permits a dynamic correlation. A constant correlation between cash and futures was imposed in previous research employing GARCH estimation in order to ensure positive definiteness of the estimated covariance matrix. This assumption is relaxed in this study by using the positive definite parameterization proposed by Engle and Kroner (1995), commonly referred to as the BEKK model. The model used in this study also incorporates asymmetries in the volatility response to return shocks of different sign but similar magnitude. Such asymmetries are analogous to the so-called "leverage effect" documented by Black (1976), Christie (1982), French, Schwert, and Stambaugh (1987), and others in studies on stock market data. The sign and size bias tests of Engle and Ng (1993) are used to provide evidence that negative return shocks have a greater impact on volatility of Canadian bankers' acceptances than positive return shocks of the same magnitude. Consequently, the standard GARCH model underpredicts the volatility response to negative shocks and overpredicts the volatility response to positive shocks. This feature of the data is incorporated into conditional hedge ratio forecasts by modeling the covariance process with the Glosten, Jaganathan, and Runkle (1993) specification. The results show that accounting for sign bias enhances the model's explanatory power. The use of statistical tests and utility comparisons corroborates the growing evidence that dynamic hedging is superior to static hedging from a statistical and a welfare standpoint, and also that further hedging improvements may be achieved by accounting for the asymmetry of the volatility response to shocks in hedge ratio forecasts.

TIME-DEPENDENT OPTIMAL HEDGE RATIOS

Consider a two-period world in which an agent's initial endowment, W_{t-1}, is fully invested in a financial instrument.[3] Denote η_{t-1} as the

[3]This is a multiperiod problem where the one-period framework is chosen for expositional convenience. A sufficient condition for a multiperiod problem to reduce to a one-period problem is that investors exhibit time-independent preferences. See Ingersoll (1987) for a discussion of this and related issues.

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investor's unit spot holding and s_{t-1} as the instrument's price at the beginning of the period. The investor may also trade futures contracts on this instrument. Let f_{t-1} represent the initial futures price for one unit of the financial security to be delivered at time T and γ_{t-1} represent the number of futures contracts included in the investor's portfolio. Since futures positions involve no initial wealth, the investor's initial portfolio is described by the following.

$$W_{t-1} = \eta_{t-1} s_{t-1} \tag{1}$$

Both cash and futures holdings are liquidated at the end of the period. Define $\tilde{s}_t$ and $\tilde{f}_t$ as the stochastic cash and futures prices as of time t. Conditional on all relevant information available at time $t-1$, Ω_{t-1}, the expected dollar return on the portfolio over the period is equal to

$$E(\tilde{R}_t \mid \Omega_{t-1}) = \eta_{t-1}(E(\tilde{s}_t \mid \Omega_{t-1}) - s_{t-1}) + \gamma_{t-1}(E(\tilde{f}_t \mid \Omega_{t-1}) - f_{t-1}) \tag{2}$$

Its variance, conditional on the same information set, is equal to

$$\begin{aligned} \mathrm{Var}(\tilde{R}_t \mid \Omega_{t-1}) = {} & \eta_{t-1}^2 \mathrm{Var}(\Delta \tilde{s} \mid \Omega_{t-1}) + \gamma_{t-1}^2 \mathrm{Var}(\Delta \tilde{f} \mid \Omega_{t-1}) \\ & + 2\eta_{t-1}\gamma_{t-1} \mathrm{Cov}(\Delta \tilde{s}, \Delta \tilde{f} \mid \Omega_{t-1}) \end{aligned} \tag{3}$$

where $\Delta \tilde{s}$ and $\Delta \tilde{f}$ correspond to the random change in the cash and futures price, from $t-1$ to t.

Assume that the investor's preferences are defined over the first two moments of the portfolio's terminal value distribution.[4] At time t, the investor's problem is described as

$$\underset{\gamma}{\mathrm{Max}} [E(\tilde{R}_t \mid \Omega_{t-1}) - \Psi \,\mathrm{Var}(\tilde{R}_t \mid \Omega_{t-1})] \tag{4}$$

where $\Psi > 0$ is the risk aversion parameter. The solution to (4) yields the optimal demand for futures contracts, conditional on information available at $t-1$, as

$$\gamma_{t-1}^* = \frac{\eta_{t-1} \mathrm{Cov}(\Delta \tilde{s}, \Delta \tilde{f} \mid \Omega_{t-1})}{\mathrm{Var}(\Delta \tilde{f} \mid \Omega_{t-1})} - \frac{E(\tilde{f}_t \mid \Omega_{t-1}) - f_{t-1}}{\Psi \,\mathrm{Var}(\Delta \tilde{f} \mid \Omega_{t-1})} \tag{5}$$

The optimal demand for futures contracts in (5) consists of two distinct components. The first term is the conditional variance-minimizing hedge ratio. The second is the speculative demand for futures contracts

[4]The quadratic utility assumption is retained for its simplicity. This modeling strategy has been used by many previous authors, including Johnson (1960), Stein (1961), and Anderson and Danthine (1981).

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which is increasing in the expected change in the futures price over the period, and decreasing in risk aversion and in the variance of the contract. Both the conditional variance-minimizing hedge ratio and the speculative demand are time-varying, responding to new information reaching the market. If futures prices follow a martingale, $E(\tilde{f}_t \mid \Omega_{t-1}) = f_{t-1}$, speculative demand is zero and (5) collapses to its first term represented by the conditional variance-minimizing hedge ratio. The martingale assumption for commodity and financial futures prices is the subject of much debate in the literature, and will be maintained here for expositional convenience. The conditional variance-minimizing hedge ratio nests the constant (OLS) hedge model as a special case and provides a basis for comparing dynamic and static hedging strategies.

DATA

The futures data for this study consist of Wednesday settlement prices between March 7, 1990 and March 30, 1994 for the nearby three-month Canadian bankers' acceptances (BAX) futures contract and is provided by the Montreal Exchange.[5] There are five holidays occurring on Wednesdays during the sample period. Thursday prices are used in these cases. Cash quotes for three-month Canadian bankers' acceptances are used as the underlying spot position. These data are collected daily around 3:00 PM (EST/EDT) by the bank of Canada and represent an average mid-market quote obtained from a sample of Canadian financial institutions. The sample of cash and futures contains a total of 213 observations. Concerns about asynchronous futures and spot prices are minimal since trading in the BAX contract ends at 3:00 PM (EST/EDT). It is assumed that the hedger rolls over the hedge into

[5]The BAX contract was introduced by the Montreal Exchange on April 16, 1988 in order to provide money market participants a viable domestic tool for hedging short-term interest rate risk exposure. The BAX contract is the Canadian counterpart to the Eurodollar time deposit futures trading at the Chicago Mercantile Exchange's International Monetary Market. In Canada, the BA rate is used as the reference rate for domestic interest rate swaps. More than 40% of its open interest originates in the U.S. and Europe. The BAX contract design is almost identical to the Eurodollar time deposit futures contract traded at the Chicago Board of Trade's International Monetary Market (IMM). Its underlying position is C$1,000,000 in Canadian bankers' acceptances with a maturity of three months. Prices are quoted as an index which is equal to 100 minus the yield on three-month Canadian bankers' acceptances. The contract is settled in cash, has no daily price limits, and entails minimum price fluctuations equal to 1 basis point worth $25 each. The BAX trades with a quarterly expiration cycle of March, June, September, and December, at maturities of up to two years. Trading ends at 10:00 AM (EST/EDT) on the second London (United Kingdom) banking day prior to the third Wednesday of the contract month and final settlement occurs on the business day following the last trading day. The contract is cleared by Trans-Canada Options Inc.

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the next contract maturity one week before the nearby contract expires. There is sufficient liquidity in the nearby contract up to a few days before its maturity date to justify such a rollover policy.

PRELIMINARY ANALYSIS

Many financial time-series exhibit unit roots. Table Ia reports the Phillips and Perron (1988) test statistics for a truncation of lag of four. The null hypothesis that unit roots exist in the natural logarithm of price levels for both cash Canadian banker's acceptances and BAX futures is not rejected. The null is rejected, however, when the spot and futures series are first-differenced. Even though first-differences are stationary, proceeding with estimation on first-differenced series may not be appropriate if the spot and futures follow a common stochastic trend. Engel and Granger (1987) show that if two series are cointegrated, first-differencing the data imposes too many unit roots on the system, and standard inferences procedures are invalid. The null hypothesis that spot BA and BAX futures log-price levels are not cointegrated is rejected based on the augmented Engle and Granger (1987) test. Cash and futures prices cannot be cointegrated, strictly speaking, because the basis degenerates to zero at expiration of the futures contract. The above tests, therefore, may reveal that price changes are somewhat predictable by past futures premia, $\ln(\tilde{f}_{t-1}) - \ln(\tilde{s}_{t-1})$.

Table Ia also includes descriptive statistics on log-differenced cash BAs and BAX futures series multiplied by 100. Based on the skewness and the kurtosis reported here, both distributions appear to be leptokurtic. The highly significant Bera-Jarque statistics reveal significant

Table Ia
Descriptive Statistics

	PP(4)	*EG*	*Skewness*	*Kurtosis*	*BJ*	*Q*	*Q^2*	*ARCH(5)*
Cash	−2.14	−5.43	−5.43	46.75	15453.06	40.55	25.89	3.55
	(−3.43)	(−4.96)	(0.00)	(0.00)	(0.00)	(0.02)	(0.36)	(0.74)
Futures	−1.84		−0.22	6.31	268.19	50.97	54.51	20.92
	(−3.43)		(0.26)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

This table presents preestimation diagnostics for Canadian bankers' acceptances and their corresponding futures (BAX) contract for the period March 7, 1990 to April 7, 1993 (N = 161).
PP(4) is the Phillips-Perron (1988) test statistic for unit roots with a truncation lag of 4. The null hypothesis is that a unit root exists. EG is the augmented Engel-Granger (1987) test statistic for cointegration of cash and futures; the null hypothesis is no cointegration. Also provided is the Bera-Jarque statistic, Q and Q^2 Ljung-Box statistics for 24 lags of the covariances of the residuals and squared residuals, and the ARCH(5) statistic for autoregressive conditional heteroskedasticity.
PP(4) and EG are computed from the natural logarithm of Wednesday prices. The other statistics are based on log differences. The 1% critical values appearing in parentheses for PP(4) and EG are taken from Davidson and MacKinnon (1993) Tables 20.1 and 20.2. P-values are provided in parentheses for the other statistics.

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Table Ib
Sign and Size Bias Tests

	Cash		*Futures*	
	Coefficient	*p-Value*	*Coefficient*	*p-Value*
Individual tests				
Sign bias	2.268	0.048	0.191	0.542
Negative sign bias	−4.235	0.006	−1.699	0.016
Positive sign bias	−1.919	0.572	0.605	0.419
Joint tests				
Sign bias	1.490	0.245	0.004	0.992
Negative sign bias	−3.629	0.024	−1.960	0.016
Positive sign bias	0.731	0.839	1.120	0.190

This table presents the results of tests for the asymmetric effect of new information on the volatility of cash and futures returns as developed by Engel and Ng (1993).

Sign bias: $\bar{v}_t^2 = a + b\bar{S}_{t-1}^- + \bar{e}_t$, $\bar{v}_t^2 = \bar{\varepsilon}_t/\sigma$, $\bar{S}_{t-1}^- = \begin{cases} 1 & \text{if } \bar{\varepsilon}_{t-1} < 0 \\ 0 & \text{otherwise} \end{cases}$

Negative sign bias: $\bar{v}_t^2 = a + b\bar{S}_{t-1}^-\bar{\varepsilon}_{t-1} + \bar{e}_t$

Positive sign bias: $\bar{v}_t^2 = a + b\bar{S}_{t-1}^+\bar{\varepsilon}_{t-1} + \bar{e}_t$, $\bar{S}_{t-1}^+ = 1 - \bar{S}_{t-1}^-$

Joint test: $\bar{v}_t^2 = a + b_1\bar{S}_{t-1}^- + b_2\bar{S}_{t-1}^-\bar{\varepsilon}_{t-1} + b_3\bar{S}_{t-1}^+\bar{\varepsilon}_{t-1} + \bar{e}_t$

departures from normality for both series. Moreover, these departures from normality are more acute for the cash market data. The Ljung-Box Q statistics [Ljung and Box (1978)] for 24 lags of the autocorrelation function are significant at any reasonable confidence level, indicating the presence of serial correlation in cash and spot returns. The Q^2 test performed on the squared log-differences reveals the presence of heteroscedasticity in the futures series but not in the spot. This conclusion is supported by the ARCH(5) test for autoregressive conditional heteroscedasticity, but a more powerful test of conditional heteroscedasticity is carried out in the context of maximum likelihood estimation of the GARCH model presented below.

Previous studies of time-varying hedge models assume that volatility is equally affected by positive and negative return shocks to spot and futures. This assumption may be appropriate for certain instruments but not others.[6] It is generally accepted that interest rate volatility is proportionately lower when rates are low and proportionately higher when rates are high. Most models of the short-term interest rate process

[6]For instance, Kroner and Sultan (1993) find no evidence of asymmetries in the volatility response to positive and negative shocks and explain that the two-sided nature of currency markets would preclude such asymmetries.

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incorporate this feature.[7] By virtue of the quoting conventions used in the cash BA and BAX futures markets, one expects positive return shocks to have a smaller impact on volatility than negative return shocks of similar magnitude. Table Ib reports results for the sign bias test, the negative size bias test, as well as the positive size bias test of Engle and Ng (1993) for log-differenced spot and futures series times 100. The results document a significant negative size bias for both series under individual and joint testing as well as a significant sign bias for the spot in an individual test. This diagnostic evidence corroborates the conjecture and suggests the inclusion of asymmetric components in the conditional volatility models.

ECONOMETRIC METHODS

Implementation of hedge ratio estimation requires modeling the first two moments of the joint distribution of cash and futures returns. The model below postulates a linear process for both cash and futures prices which consists of a predictable component conditional on the information available at time $t - 1$ and a random shock component which has an expected value of zero and is serially uncorrelated conditional on the information set. The process is represented as follows

$$\tilde{s}_t = E(\tilde{s}_t \mid \Omega_{t-1}) + \tilde{\varepsilon}_{st} \tag{6}$$

$$\tilde{f}_t = E(\tilde{f}_t \mid \Omega_{t-1}) + \tilde{\varepsilon}_{ft} \tag{7}$$

where the prediction errors are assumed to be conditionally uncorrelated but their conditional covariances may change over time in response to shocks to the economy. The information set available to all investors at the beginning of the period may contain a constant, lagged prices, and other weakly exogenous variables.

Defining $\tilde{y}_t$ as a vector of log-differenced spot and futures prices multiplied by 100, the following bivariate system with a GARCH(1,1) conditional covariance matrix is estimated.

$$\tilde{y}_t = \alpha_0 + \alpha_1(\ln(\tilde{s}_{t-1}) - \ln(f_{t-1})) + \theta\varepsilon_{t-1} + \tilde{\varepsilon}_t \tag{8}$$

[7]For instance, consider the geometric Brownian motion process postulated by Dothan (1978), the square-root process proposed by Cox, Ingersole, and Ross (1985), and the double-square-root process introduced by Longstaff (1989). The well-known Ornstein-Uhlenbeck process used by Vasicek (1977) is essentially homoscedastic and, hence, allows the short rate to become negative. For recent empirical evidence on alternative specifications of the short rate process, see Chan, Karolyi, Longstaff, and Saunders (1992), Gagnon, Morgan, and Neave (1993), and Brenner, Harjes, and Kroner (1994).

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$$\tilde{\varepsilon}_t \mid \Omega_{t-1} = t(0, H_t, v) \tag{9}$$

$$H_t = C'C + A'\tilde{\varepsilon}_{t-1}\tilde{\varepsilon}_{t-1}'A + B'H_{t-1}B + D'\tilde{u}_{t-1}\tilde{u}_{t-1}'D \tag{10}$$

The conditional mean eq. (8) includes a constant, the negative of the lagged futures premium, and a moving average term to capture the autocorrelation reported in Table Ia. The conditional covariance matrix for the error vector allows for asymmetric ARCH [Glosten, Jaganathan, and Runkle (1993)] by adding $\tilde{u}_t = \min[0, \tilde{\varepsilon}_t]$. This matrix parameterization in (10) was proposed by Engle and Kroner (1995) and is commonly referred to as the BEKK model. Its main attraction over alternative parameterizations, such as the more traditional diagonal representation of Bollerslev, Engle, and Wooldridge (1988), is that it ensures positive definiteness of the conditional covariance matrix, H_t, under very weak conditions. The conditional Student t distribution is used to represent the distribution of returns shocks to account for the fat tails which characterize their conditional distributions.[8]

MAXIMUM LIKELIHOOD ESTIMATION

Table IIa reports maximum likelihood estimates of the conditional means and covariance matrix of cash and futures returns defined in eqs. (8)–(10). The estimation is carried out using the Broyden, Fletcher, Goldfarb, and Shanno (BFGS) algorithm. The right-hand side panel of the table contains estimates for what is referred to as the *general* model, because its conditional mean includes the negative of the lagged futures premium and a moving average term to account for serial correlation. The left-hand side panel contains estimates for the simpler *dynamic* model which is nested within the *general* model and is obtained by omitting the futures premium and moving average adjustments. Both dynamic models include GJR asymmetric effects.

Table IIa reveals that the GARCH(1,1) specification provides a good description of the joint distribution of spot BA and BAX futures returns. Both the spot and the futures returns series exhibit significant ARCH and GARCH components evidenced by the significance of matrix A and B elements [cf. eq. (10)]. The conditional distributions for the error terms exhibit fat tails as reflected by the estimated number of degrees of freedom, v, which is equal to 2.868 for the general and 2.861 for the dynamic model. One expects the estimated degrees of freedom

[8] Estimates of this model under the assumption of conditional normality reveal significant violations of the normality assumption in postestimation diagnostics. These results are not reported here, but are available from the authors upon request.

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TABLE IIa
Maximum Likelihood Estimates

	Dynamic Model N = 160, df = 142, Log-Likelihood = 296.73				General Model N = 160, df = 138, Log-Likelihood = 309.79			
	Coefficient	Standard Error	t-Statistic	p-Value	Coefficient	Standard Error	t-Statistic	p-Value
α_{01}	0.093	0.007	12.677	0.000	0.085	0.014	6.289	0.000
α_{02}	0.063	0.012	5.391	0.000	0.112	0.020	5.643	0.000
α_{11}	—	—	—	—	−0.019	0.034	−0.562	0.574
α_{12}	—	—	—	—	0.213	0.062	3.446	0.001
θ_1	—	—	—	—	0.171	0.063	2.718	0.007
θ_2	—	—	—	—	0.216	0.068	3.182	0.001
c_{11}	0.073	0.020	3.745	0.000	0.078	0.019	4.180	0.000
c_{12}	0.021	0.036	0.589	0.556	0.054	0.035	1.534	0.125
c_{22}	0.125	0.028	4.441	0.000	0.142	0.031	4.589	0.000
a_{11}	0.588	0.199	2.951	0.003	0.616	0.223	2.761	0.006
a_{12}	−0.370	0.166	−2.232	0.026	−0.187	0.152	−1.231	0.218
a_{21}	−0.071	0.097	−0.735	0.462	−0.005	0.084	−0.063	0.950
a_{22}	0.824	0.221	3.725	0.000	0.798	0.241	3.310	0.001
b_{11}	0.625	0.083	7.514	0.000	0.587	0.114	5.171	0.000
b_{12}	0.365	0.130	2.807	0.005	0.353	0.147	2.403	0.016
b_{21}	0.094	0.065	1.447	0.148	0.065	0.080	0.807	0.419
b_{22}	0.565	0.100	5.675	0.000	0.461	0.143	3.233	0.001
d_{11}	−0.103	0.304	−0.338	0.735	0.377	0.366	1.030	0.303
d_{12}	0.008	0.214	0.038	0.970	0.221	0.191	1.156	0.248
d_{21}	−0.545	0.186	−2.931	0.003	0.531	0.196	2.708	0.007
d_{22}	−0.007	0.294	−0.024	0.981	−0.130	0.266	−0.490	0.624
v	2.868	0.449	6.384	0.000	2.861	0.480	5.964	0.000

This table reports joint maximum likelihood estimates of the conditional means and covariance matrix of returns on three-month Canadian bankers' acceptances (BAs) and their corresponding futures contract (BAX) for the following bivariate GARCH (1,1) specification

$$\tilde{y}_t = \alpha_0 + \alpha_1(\ln(\tilde{s}_{t-1}) - \ln(f_{t-1})) + \theta\varepsilon_{t-1} + \tilde{\varepsilon}_t$$

$$\tilde{\varepsilon}_t \mid \Omega_{t-1} = t(0, H_t, v)$$

$$H_t = C'C + A'\tilde{\varepsilon}_{t-1}\tilde{\varepsilon}'_{t-1}A + B'H_{t-1}B + D'\tilde{u}_{t-1}\tilde{u}'_{t-1}D$$

The index 1 refers to the spot BA position and 2 refers to the futures contract.
The data are weekly, log-differenced prices covering the period Wednesday, March 7, 1990 to Wednesday, April 7, 1993 for 160 observations.
The right-hand side of the table contains estimates for the full system (the general model), and the left-hand side restricts the α_1 and θ parameter vectors of the mean equations to zero (dynamic model).

to be considerably bigger for the conditional normal distribution to be a sensible distributional assumption. The log-likellihood for the general model is equal to 309.79, which is significantly higher at any reasonable level, than the log-likelihood of 296.73 for the dynamic model.

The likelihood ratio tests reported in Table IIb show that various restrictions imposed on the general model result in models with sig-

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Table IIb
Tests on Model Restrictions

	Log-Likelihood Value	*Likelihood Ratio*	*p-Value*	df
1. General model	309.8			138
2. $D = 0$	298.4	22.7	0.0	142
3. Dynamic model ($\alpha = \theta = 0$)	296.7	26.1	0.0	142
4. Diagonal A, B, D	285.5	48.6	0.0	148
5. $A = B = D = 0$	256.5	106.5	0.0	154

This table presents likelihood ratio tests showing that various restrictions on the general model result in significantly less explanatory power.

nificantly less explanatory power.[9] Restriction 2 implies the symmetric GARCH(1,1) model in which positive and negative innovations of the same magnitude have the same impact on volatility. This model is rejected in favor of the general model based on the LR test.[10] According to the estimates shown in Table IIa, the asymmetry is driven by the cross term d_{21}. The third model restriction sets the futures premium and the moving average terms to zero and again the restricted model has considerably less explanatory power than its more general counterpart. The same conclusion is reached with restriction 4 (imposed by previous authors) where the off-diagonal elements of the A, and B are set to zero. Finally, the OLS model in restriction 5 is tested and rejected as well.

The evidence presented in this section leads one to expect hedging exercises based upon the general model to outperform those based upon the dynamic model, and those based on the dynamic model to yield greater variance reduction than those resting on the OLS model. The next section turns to formal tests of this proposition with an investigation of within-sample and out-of-sample performance of the two dynamic models and the two popular static hedging models.

HEDGING EFFECTIVENESS

Variance Reduction

Hedging efficiency is typically measured as the percentage reduction in the portfolio's variance achieved by hedging under a particular model. In this spirit, Table III presents in-sample and out-of-sample comparisons

[9] The same battery of comparisons between the dynamic model and nested alternatives yield qualitatively similar results.
[10] This restriction is also imposed on the dynamic model and the test results are qualitatively similar.

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TABLE III

Hedging Effectiveness

	In-Sample		*Out-of-Sample*	
	Variance	*Change (%)*	*Variance*	*Change (%)*
Unhedged	0.1662		0.0543	
Naive	0.1085	−34.73	0.0367	−34.30
OLS	0.1015	−6.43	0.0231	−34.63
Dynamic	0.0924	−9.01	0.0229	−1.87
General	0.0949	2.76	0.0231	0.91

This table compares the hedging effectiveness of two dynamic hedging models called dynamic and general, and two static hedging models called naive and OLS. Hedging effectiveness is measured as percentage reduction in portfolio variance. The in-sample period begins March 7, 1990 and runs for 160 weeks to April 7, 1993. The out-of-sample period begins on April 14, 1993 and runs for 51 weeks to March 30, 1994.

The dynamic models correspond to the bivariate BEKK parameterization described in Table II and eqs. (8)–(10) in the body of the article. The OLS model corresponds to a hedge ratio estimated by at least squares regression of log-differenced spot on futures. The naive model uses a hedge ratio of one.

of the variance reduction achieved with the general model and dynamic model, the OLS model, and the naive model. The within-sample estimation is carried out on the first 160 observations starting on March 7, 1990 and ending on April 7, 1993. The out-of-sample experiments involve the last 51 weeks of the sample running to March 30, 1994. These out-of-sample experiments involve one-period-ahead forecasts of the conditional hedge ratios. At the beginning of each week in the subsample, the model is estimated with all previous observations in the sample up to the current week. The conditional hedge ratio is then computed from the relevant elements of the updated H_t matrix, and the hedge is simulated for that week. This procedure is repeated for each of the last 51 weeks of the sample.

As shown in Table III, the rank-ordering of models is the same within-sample as out-of-sample with dynamic model achieving the highest percentage of variance reduction, followed by the general model, the OLS, finally the naive model. Reductions in risk relative to the OLS hedge achieved with the dynamic model and general model (9% and 6%) compare well with the findings of Kroner and Sultan (1993) for foreign currencies.[11]

The general model was expected to outperform the dynamic model based on the former model's statistical superiority in explaining the dynamics of spot and futures returns (cf. Table II). It is possible that the dynamic model actually performs better because the general model overfits the data. The marginally poorer performance of the general

[11]The general and dynamic models also outperform the restricted models described in Table IIb.

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model relative to the dynamic model is not due to a confounding interaction between the lagged futures premium and moving average adjustments in the mean equations, because the performance ranking is unchanged when either adjustment is omitted from the general model. The evidence, therefore, suggests that the dynamic model is a parsimonious representation for the purposes of hedging effectiveness. Also noteworthy is the superior hedging performance of the OLS model over the naive model, both within and out-of-sample. This evidence contrasts with the widely-held belief among practitioners and academics that the naive approach yields greater variance reduction than the OLS model for Eurodollar and similar instruments.

The conclusion to be drawn here is that the dynamic models lead to greater variance reduction than their counterparts, the naive and OLS models. The next issue addressed is whether the incremental reduction in variance achieved with the general model and the dynamic model over the static models is significant from a welfare standpoint.

Measuring Economic Significance

To assess whether the efficiency gains brought about by the dynamic hedging models are economically meaningful, investor preferences have to be taken into consideration. As in Kroner and Sultan (1993), the dynamic models will be considered to be economically superior to the OLS model if they result in higher expected utility net of transactions costs than the static models.

In the presence of transactions costs, the investor's problem represented by eq. (4) can be easily amended to incorporate the cost of rebalancing the hedge at every period. Table IVa shows the degree of risk aversion, ψ, for a mean-variance utility maximizer to be exactly indifferent between the dynamic hedging strategy and the constant hedging strategy as a function of the level of transactions costs. At the Montreal exchange, the round-trip transactions cost for one BAX futures contract is \$1 for floor traders, \$3 for Exchange members, and about \$5 for retail customers.

Table IVa is based upon the percentage variance reduction reported in Table III. For instance, assuming a \$5 round-trip transaction cost, an investor with risk aversion coefficient greater than 0.055 (0.76) would have derived higher average utility within-sample by following the dynamic model (general model) rather than holding the hedge constant (OLS). The range of values for ψ obtained in this table is well below the empirical estimates commonly reported in the literature [cf. references

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Table IVa
Average Utility Comparison

	In-Sample		*Out-of-Sample*	
Cost	*General*	*Dynamic*	*Dynamic*	*General*
$1	0.015	0.011	0.229	0.441
$3	0.045	0.033	0.688	1.322
$5	0.076	0.055	1.146	2.203

This table shows the degree of risk aversion, ψ, for which a mean-variance utility maximizer is indifferent between a constant hedge and a dynamic hedge given the round-trip, transactions cost for a one-unit spot position.

cited by Kroner and Sultan (1993, footnote 6, p. 545)]. The implication is that one does not need to be very risk averse to benefit from the dynamic model or the general model. It follows also from comparisons of the two dynamic models that higher utility would result from the dynamic model than from the general model.

The above analysis actually understates the economic superiority of the two dynamic strategies because it is conducted under the assumption that the investor rebalances the portfolio at the beginning of every period. In a more realistic setting, one would choose to rebalance the hedge only at points in time when welfare gains are expected. This situation is examined in Table IVb. For instance, an investor with $\psi = 3$ and facing a round-trip transaction cost of $5 would have rebalanced the portfolio on 32 occasions under the dynamic model during the 51 week out-of-sample period. This compares most favorably to the rebalancing frequencies documented by Kroner and Sultan (1993) for FX hedges.

TABLE IVb
Total Expected Utility Comparison

	Cost					
	$1		*$3*		*$5*	
	Utility	*Rebalancings*	*Utility*	*Rebalancings*	*Utility*	*Rebalancings*
OLS	−13.833	2	−13.834	1	−13.834	1
General	−5.304	36	−5.311	32	−5.317	27
Dynamic	−6.210	42	−6.218	34	−6.224	32

This table presents an out-of-sample comparison of the total expected utility derived when discretionary portfolio rebalancing is recognized. The investor rebalances, applying a new hedge ratio, and incurs the rebalancing cost only if mean-variance utility is greater; otherwise, the most recent hedge ratio is maintained and no cost is incurred. It is assumed that the degree of risk aversion, ψ, is 3. Floor traders at the Montreal Exchange incur a round-trip cost of $1 per $1,000,000 contract or 0.0001%, cost to members is about $3 and to clients about $5.

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Total expected utility levels realized with each model as a function of transactions costs are reported along with the number of times the hedge would have been rebalanced. The dynamic models produce higher expected utility than the OLS model as expected from the previous analysis. The expected utility level associated with the general model is higher than that for the dynamic model but due to an overfitting problem which seems to tax the general model's forecasting ability, the dynamic model delivers the highest average utility level (cf. Table IVa).

CONCLUSION

This article is concerned with dynamic hedging of short-term interest rate risk represented by positions in bankers' acceptances. Two dynamic models employing the GARCH framework are examined. These models differ from other models proposed in the literature by incorporating asymmetric shocks to volatility and by using the BEKK positive-definite parameterization which allows the conditional correlation between spot and futures returns, as well as their own variances, to vary over time. The evidence supports the asymmetric GJR-GARCH(1,1) model as a description of the joint dynamics of spot and futures returns. In-sample and out-of-sample comparisons of the performance of the two dynamic models to the well-known OLS hedging strategy as well as to the widely used duration-based approach reveal that the two dynamic models perform significantly better than the two static strategies, both statistically and economically.

The in-sample and out-of-sample experiments also suggest that the OLS hedge ratio model has greater risk-reduction potential than the duration-based approach. This result represents a departure from accepted wisdom because the duration-based model is generally considered to be the best approach to use for the hedging of short-term, and indeed long-term, interest rate risk.

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